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# On the theory of a dicluster-ion beam propagation through a thin foil

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**Abstract.** The propagation of a dicluster-ion (a pair of closely situated ions) beam through a system consisting of two thin foils is considered in this paper. The effect of wake fields on the distribution of ions flying out from a foil in the initial beam direction is analysed, as well as their effect on the number of accelerated and retarded ions registered by the detector. The increase in the measurement efficiency of the wake fields is discussed.

## 1. Introduction

The potential of a fast charged particle moving through a material medium is known to be different from the spherically symmetric Debye or Coulomb potential (Vager and Gemmell 1976, Vager *et al* 1976). The difference has been revealed in experiments with molecular ion beams ($H_2^+$, $He_2^+$, $D_2^+$, $D_3^+$, $HeH^+$, $HeD^+$, $CH^+$, $OH^+$, etc) passing through thin foils (Ag, Au, C, Al, etc). In the numerous experiments carried out since 1965 (Vager and Gemmell 1976, Vager *et al* 1976, Gemmell 1980, Remillieux 1980), a beam of molecular ions was accelerated to an energy of several megaelectronvolts per nucleon and focused on a 10–100 Å thin foil.

On colliding with the target, each molecular ion is stripped of its electrons at distances much smaller than the foil thickness, as a result of which the so-called 'Coulomb explosion' is developed, i.e. ions are scattered by the repulsion forces acting between them. Clearly, the energy spectrum, angular distribution of ions and other scattering characteristics depend on the potential created by the ions within the foil.

A peculiarity of the experiments described above was that the distance between the ions that originated after the 'Coulomb explosion' was significantly less than the wake field wavelength (Gemmell 1980, Remillieux 1980). Thus, the electric field of the charged particle was measured at small distances in comparison with the wake wavelength. The theory corresponding to this case was developed by Kagan *et al* (1978).

Kumbartzki *et al* (1982) have considered the propagation of molecular ion beam through the two-foil system. In this experiment, the molecular ion beam passed through a very thin foil of thickness $a_0$, the only purpose of which was to strip the molecular ions of its electrons. After such ionization, the molecular ion decayed into separate ions, due to a 'Coulomb explosion', which—upon flying out from the foil—move in a vacuum (in the form of a dicluster of two particles of like charge). The second foil of thickness $a$ was

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placed after the first, at a distance $d$. The motion of ions in the gap $d$ is determined by a Coulomb potential. By selection of the distance $d$, one may make the ions interact in the second foil, under the effect of a long-range wake potential.

Gorbunov and Nersissian (1993) considered the dynamics of the ionic dicluster propagation through a thin foil, taking into account surface effects.

Recently, interest in these phenomena has grown significantly, particularly in the context of the ionic thermonuclear fusion problem (Avanzo *et al* 1992, 1993, Zinamon 1993).

In the presented paper, the statistical theory of propagation of the dicluster-ion beam through the layer of a plasma-like medium (i.e. a medium containing free electrons) is constructed, according to the experiment of Kumbartzki *et al* (1982). Also the conditions under which the influence of wake fields is most efficient are considered.

The paper outline is as follows. In section 2, general expressions are obtained for the distribution functions of ions originating in the 'Coulomb explosion' of the initial molecular beam (Kumbartzki *et al* 1982) and being either retarded or accelerated with respect to the beam (these ions will be called 'retarded' and 'accelerated' ions in the paper) in the Liouville equation approximation (Krall and Trivelpiece 1975). General expressions for the number of retarded and accelerated ions hitting the detector are also found. The generality of the results obtained is that they are independent of the method of ionization of the molecular ion beam but they are dependent on the trajectories of motion of the dicluster ions. In section 3, the general expressions obtained are applied to the calculation of the distribution function of retarded and accelerated ions emitted from the foil in the direction of motion of the primary beam. In section 4, the numbers of retarded and accelerated ions hitting the detector are found.

## 2. General considerations

Consider a beam of molecular ions having velocity $\boldsymbol{u}_0$ directed along the $z$ axis, which is normal to the surface of a thin foil. Let the thin foil form an ionic dicluster of molecular ions, with masses $m_1$ and $m_2$, and charges $Q_1$ and $Q_2$, respectively.

We shall describe the dicluster beam by a two-particle distribution function $f(\boldsymbol{r}_1, \boldsymbol{r}_2; \boldsymbol{u}_1, \boldsymbol{u}_2; t)$, where

$$f(\boldsymbol{r}_1, \boldsymbol{r}_2; \boldsymbol{u}_1, \boldsymbol{u}_2; t)\, \mathrm{d}\boldsymbol{r}_1\, \mathrm{d}\boldsymbol{r}_2\, \mathrm{d}\boldsymbol{u}_1\, \mathrm{d}\boldsymbol{u}_2$$

is the number of particles having masses $m_1$, $m_2$ and charges $Q_1$, $Q_2$ in a differential phase volume at a time instant $t$. This function satisfies the Liouville equation (Krall and Trivelpiece 1975)

$$\frac{\partial f}{\partial t} + \boldsymbol{u}_1 \cdot \frac{\partial f}{\partial \boldsymbol{r}_1} + \boldsymbol{u}_2 \frac{\partial f}{\partial \boldsymbol{r}_2} + \frac{\boldsymbol{F}_1}{m_1} \frac{\partial f}{\partial \boldsymbol{u}_1} + \frac{\boldsymbol{F}_2}{m_2} \frac{\partial f}{\partial \boldsymbol{u}_2} = 0 \tag{2.1}$$

where $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ are forces acting at dicluster ions with masses $m_1$ and $m_2$, respectively.

The Liouville equation (2.1) has the following characteristic equations of motion for the ions:

$$\begin{aligned} &\boldsymbol{F}_1 = m_1 \dot{\boldsymbol{u}}_1 \qquad \boldsymbol{F}_2 = m_2 \dot{\boldsymbol{u}}_2 \\ &\boldsymbol{u}_1 = \dot{\boldsymbol{r}}_1 \qquad \boldsymbol{u}_2 = \dot{\boldsymbol{r}}_2 \end{aligned} \tag{2.2}$$

with the initial conditions (at $t = 0$) $\boldsymbol{r}_1 = \boldsymbol{r}_{10}$, $\boldsymbol{r}_2 = \boldsymbol{r}_{20}$, $\boldsymbol{u}_1 = \boldsymbol{u}_2 = \boldsymbol{u}_0$.

It is convenient to describe the distribution function at a time $t = 0$ in the coordinate system with variables

$$\boldsymbol{r}_0 = \boldsymbol{r}_{10} - \boldsymbol{r}_{20} \qquad \boldsymbol{R}_0 = \frac{m_1 \boldsymbol{r}_{10} + m_2 \boldsymbol{r}_{20}}{m_1 + m_2}. \tag{2.3}$$

Then, denoting $f(\boldsymbol{r}_1, \boldsymbol{r}_2; \boldsymbol{u}_1, \boldsymbol{u}_2; t = 0)$ as $f_0(\Gamma_0)$, where $\Gamma_0$ is the set of variables $\boldsymbol{r}_0$ and $\boldsymbol{R}_0$, we may present the solution of equation (2.1) in the following form:

$$f(\Gamma, t) = \int \mathrm{d}\Gamma_0\, f_0(\Gamma_0)\delta(\boldsymbol{r}_1 - \boldsymbol{r}_1(\Gamma_0, t))\delta(\boldsymbol{r}_2 - \boldsymbol{r}_2(\Gamma_0, t))$$
$$\times\delta(\boldsymbol{u}_1 - \boldsymbol{u}_1(\Gamma_0, t))\delta(\boldsymbol{u}_2 - \boldsymbol{u}_2(\Gamma_0, t)). \tag{2.4}$$

Here $\Gamma$ is the set of variables $\boldsymbol{r}_1$, $\boldsymbol{r}_2$, $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$, $\mathrm{d}\Gamma_0 = \mathrm{d}\boldsymbol{r}_0\boldsymbol{R}_0$ is the differential volume of the phase space, while $\boldsymbol{r}_j(\Gamma_0, t)$ and $\boldsymbol{u}_j(\Gamma_0, t)$ with $j = 1, 2$ are solutions to the characteristic equations (2.2).

We shall also normalize the distribution functions $f_0(\Gamma_0)$ and $f(\Gamma, t)$ as follows:

$$\int \mathrm{d}\Gamma\, f(\Gamma, t) = \int \mathrm{d}\Gamma_0\, f_0(\Gamma_0) = 2N \tag{2.5}$$

where $\mathrm{d}\Gamma = \mathrm{d}\boldsymbol{r}_1\, \mathrm{d}\boldsymbol{r}_2\, \mathrm{d}\boldsymbol{u}_1\, \mathrm{d}\boldsymbol{u}_2$ is the differential volume of phase space, $N$ is the total number of molecular ions in the initial beam, while $2N$ is the total number of ions that were created after emergence of the beam from the foil. Integration in equations (2.4) and (2.5) is taken over the entire phase space $\Gamma_0$.

We shall isolate two integration domains $\boldsymbol{r}_0 \cdot \boldsymbol{u}_0 < 0$ and $\boldsymbol{r}_0 \cdot \boldsymbol{u}_0 > 0$ in (2.4) in order to obtain the distribution functions for accelerated and retarded particles. The first domain corresponds to a distribution function of retarded particles having mass $m_1$ and accelerated particles having mass $m_2$, while the second domain corresponds to the distribution of retarded $m_2$ particles and accelerated $m_1$ particles. Integrating these distribution functions over the variables $(\boldsymbol{r}_1, \boldsymbol{u}_1)$ and $(\boldsymbol{r}_2, \boldsymbol{u}_2)$, respectively, one may obtain the expressions for the distribution functions of particles having masses $m_2$ and $m_1$:

$$f_1^+(\boldsymbol{r}_1, \boldsymbol{u}_1, t) = \tfrac{1}{2}\int_{\boldsymbol{r}_0\cdot\boldsymbol{u}_0>0} \mathrm{d}\Gamma_0\, f_0(\Gamma_0)\delta(\boldsymbol{r}_1 - \boldsymbol{r}_1(\Gamma_0, t))\delta(\boldsymbol{u}_1 - \boldsymbol{u}_1(\Gamma_0, t)) \tag{2.6}$$

$$f_2^+(\boldsymbol{r}_2, \boldsymbol{u}_2, t) = \tfrac{1}{2}\int_{\boldsymbol{r}_0\cdot\boldsymbol{u}_0<0} \mathrm{d}\Gamma_0\, f_0(\Gamma_0)\delta(\boldsymbol{r}_2 - \boldsymbol{r}_2(\Gamma_0, t))\delta(\boldsymbol{u}_2 - \boldsymbol{u}_2(\Gamma_0, t)) \tag{2.7}$$

$$f_1^-(\boldsymbol{r}_1, \boldsymbol{u}_1, t) = \tfrac{1}{2}\int_{\boldsymbol{r}_0\cdot\boldsymbol{u}_0<0} \mathrm{d}\Gamma_0\, f_0(\Gamma_0)\delta(\boldsymbol{r}_1 - \boldsymbol{r}_1(\Gamma_0, t))\delta(\boldsymbol{u}_1 - \boldsymbol{u}_1(\Gamma_0, t)) \tag{2.8}$$

$$f_2^-(\boldsymbol{r}_2, \boldsymbol{u}_2, t) = \tfrac{1}{2}\int_{\boldsymbol{r}_0\cdot\boldsymbol{u}_0>0} \mathrm{d}\Gamma_0\, f_0(\Gamma_0)\delta(\boldsymbol{r}_2 - \boldsymbol{r}_2(\Gamma_0, t))\delta(\boldsymbol{u}_2 - \boldsymbol{u}_2(\Gamma_0, t)) \tag{2.9}$$

in which $f_1^\pm(\boldsymbol{r}_1, \boldsymbol{u}_1, t)$ and $f_2^\pm(\boldsymbol{r}_2, \boldsymbol{u}_2, t)$ correspond to accelerated and retarded particles with masses equal to $m_1$ and $m_2$, respectively. Domains of integration in (2.6)–(2.9) are determined by the inequality $\boldsymbol{r}_0 \cdot \boldsymbol{u}_0 < 0$ or $\boldsymbol{r}_0 \cdot \boldsymbol{u}_0 > 0$. The origin of the factor $\frac{1}{2}$ in equations (2.6)–(2.9) is due to the bounded character of the integration domain over the variables $(\boldsymbol{r}_1, \boldsymbol{u}_1)$ and $(\boldsymbol{r}_2, \boldsymbol{u}_2)$.

The normalization conditions for distribution functions $f_1^\pm(\boldsymbol{r}_1, \boldsymbol{u}_1, t)$ and $f_2^\pm(\boldsymbol{r}_2, \boldsymbol{u}_2, t)$ follow equations (2.6)–(2.9):

$$f\, \mathrm{d}\boldsymbol{r}_1\, \mathrm{d}\boldsymbol{u}_1[f_1^+(\boldsymbol{r}_1, \boldsymbol{u}_1, t) + f_1^-(\boldsymbol{r}_1, \boldsymbol{u}_1, t)]$$
$$= \int \mathrm{d}\boldsymbol{r}_2\, \mathrm{d}\boldsymbol{u}_2[f_2^+(\boldsymbol{r}_2, \boldsymbol{u}_2, t) + f_2^-(\boldsymbol{r}_2, \boldsymbol{u}_2, t)] = N. \tag{2.10}$$

To determine the flux densities of the accelerated and retarded particles, we shall make use of equations (2.6)–(2.9):

$$\boldsymbol{j}^+ = \tfrac{1}{2}\int_{\boldsymbol{r}_0\cdot\boldsymbol{u}_0>0} \mathrm{d}\Gamma_0\, f_0(\Gamma_0)\boldsymbol{u}_1(\Gamma_0, t)\delta(\boldsymbol{r}_1 - \boldsymbol{r}_1(\Gamma_0, t))$$

$$+\tfrac{1}{2}\int_{\boldsymbol{r}_0\cdot\boldsymbol{u}_0<0}\mathrm{d}\Gamma_0\, f_0(\Gamma_0)\boldsymbol{u}_2(\Gamma_0,t)\delta(\boldsymbol{r}_2-\boldsymbol{r}_2(\Gamma_0,t)) \tag{2.11}$$

$$\boldsymbol{j}^- = \tfrac{1}{2}\int_{\boldsymbol{r}_0\cdot\boldsymbol{u}_0<0}\mathrm{d}\Gamma_0\, f_0(\Gamma_0)\boldsymbol{u}_1(\Gamma_0,t)\delta(\boldsymbol{r}_1-\boldsymbol{r}_1(\Gamma_0,t)) + \tfrac{1}{2}\int_{\boldsymbol{r}_0\cdot\boldsymbol{u}_0>0}\mathrm{d}\Gamma_0\, f_0(\Gamma_0)\boldsymbol{u}_2(\Gamma_0,t)\delta(\boldsymbol{r}_2-\boldsymbol{r}_2(\Gamma_0,t)). \tag{2.12}$$

The first terms on the right-hand sides of these expressions correspond to fluxes of accelerated and retarded particles of mass $m_1$, while the second terms describe the same fluxes of particles of mass $m_2$.

Particle velocity distributions may be further obtained from equations (2.6)–(2.9) by integration over their coordinates:

$$f_1^{\pm}(\boldsymbol{u}_1,t) = \tfrac{1}{2}\int_{\boldsymbol{r}_0\cdot\boldsymbol{u}_0>;<0}\mathrm{d}\Gamma_0\, f_0(\Gamma_0)\delta(\boldsymbol{u}_1-\boldsymbol{u}_1(\Gamma_0,t)) \tag{2.13}$$

$$f_2^{\pm}(\boldsymbol{u}_2,t) = \tfrac{1}{2}\int_{\boldsymbol{r}_0\cdot\boldsymbol{u}_0<;>0}\mathrm{d}\Gamma_0\, f_0(\Gamma_0)\delta(\boldsymbol{u}_2-\boldsymbol{u}_2(\Gamma_0,t)). \tag{2.14}$$

Note that, at large distances from the foil, the particle velocities become constant; therefore the distribution functions of velocities should be time independent.

Let a circular detector diaphragm be positioned in the plane $z = L$ (and the foil be situated in the plane $z = 0$), centred at the $z$ axis and having radius $D$. Then integration of the fluxes $\boldsymbol{j}^+$ and $\boldsymbol{j}^-$ over the diaphragm and time will give the following expressions for the total number of accelerated and retarded ions entering the detector:

$$N^+ = \tfrac{1}{2}\int_{\boldsymbol{r}_0\cdot\boldsymbol{u}_0>0}\mathrm{d}\Gamma_0\, f_0(\Gamma_0)\eta(D-|\boldsymbol{r}_{\perp 1}(\Gamma_0,t_1)|) + \tfrac{1}{2}\int_{\boldsymbol{r}_0\cdot\boldsymbol{u}_0<0}\mathrm{d}\Gamma_0\, f_0(\Gamma_0)\eta(D-|\boldsymbol{r}_{\perp 2}(\Gamma_0,t_2)|) \tag{2.15}$$

$$N^- = \tfrac{1}{2}\int_{\boldsymbol{r}_0\cdot\boldsymbol{u}_0<0}\mathrm{d}\Gamma_0\, f_0(\Gamma_0)\eta(D-|\boldsymbol{r}_{\perp 1}(\Gamma_0,\tau_1)|) + \tfrac{1}{2}\int_{\boldsymbol{r}_0\cdot\boldsymbol{u}_0>0}\mathrm{d}\Gamma_0\, f_0(\Gamma_0)\eta(D-|\boldsymbol{r}_{\perp 2}(\Gamma_0,\tau_2)|). \tag{2.16}$$

Here $\eta(x)$ is the Heaviside function (with $\eta(0) = \frac{1}{2}$), $\boldsymbol{r}_{\perp 1}$ and $\boldsymbol{r}_{\perp 2}$ are the transverse coordinates of particles, while $t_1$, $\tau_1$ and $t_2$, $\tau_2$ are the time instants when accelerated and retarded particles, respectively, with masses $m_1$ and $m_2$, cross the plane $z = L$. The values $t_1$, $\tau_1$ and $t_2$, $\tau_2$ are determined from the following relations:

$$z_1(\Gamma_0,t_1) = z_2(\Gamma_0,\tau_2) = L \qquad \boldsymbol{r}_0\cdot\boldsymbol{u}_0 > 0 \tag{2.17}$$

$$z_1(\Gamma_0,\tau_1) = z_2(\Gamma_0,t_2) = L \qquad \boldsymbol{r}_0\cdot\boldsymbol{u}_0 < 0 \tag{2.18}$$

where $z_1(\Gamma_0,t)$ and $z_2(\Gamma_0,t)$ are the longitudinal coordinates of particles, which are different in domains $\boldsymbol{r}_0\cdot\boldsymbol{u}_0 < 0$ and $\boldsymbol{r}_0\cdot\boldsymbol{u}_0 > 0$ due to difference between the forces acting on the accelerated and the retarded particles.

Equations (2.13) and (2.14), as well as (2.15) and (2.16), are essentially simplified in the case of identical ions (i.e. when $m_1 = m_2 = m$, and $Q_1 = Q_2 = Q$). In this case, the initial distribution function $f_0(\Gamma_0)$ is symmetric with respect to the boundary $\boldsymbol{r}_0\cdot\boldsymbol{u}_0 = 0$ of two domains. Therefore the functions $f_1^+(\boldsymbol{u}_1,t)$ and $f_2^+(\boldsymbol{u}_2,t)$ are identical, as well as $f_1^-(\boldsymbol{u}_1,t)$ and $f_2^-(\boldsymbol{u}_2,t)$, and equations (2.13) and (2.14) become

$$f^+(\boldsymbol{u}_1,t) = \int_{\boldsymbol{r}_0\cdot\boldsymbol{u}_0>0}\mathrm{d}\Gamma_0\, f_0(\Gamma_0)\delta(\boldsymbol{u}_1-\boldsymbol{u}_1(\Gamma_0,t)) \tag{2.19}$$

$$f^{-}(\boldsymbol{u}_2, t) = \int_{\boldsymbol{r}_0\cdot\boldsymbol{u}_0>0} \mathrm{d}\Gamma_0\, f_0(\Gamma_0)\delta(\boldsymbol{u}_2 - \boldsymbol{u}_2(\Gamma_0, t)) \tag{2.20}$$

where $f^{+}(\boldsymbol{u}_1, t)$ and $f^{-}(\boldsymbol{u}_2, t)$ are the distribution functions for accelerated and retarded particles, respectively.

In a similar way, from (2.15) and (2.16) we have

$$N^{+} = \int_{\boldsymbol{r}_0\cdot\boldsymbol{u}_0>0} \mathrm{d}\Gamma_0\, f_0(\Gamma_0)\eta(D - |\boldsymbol{r}_{\perp 1}(\Gamma_0, t_1)|) \tag{2.21}$$

$$N^{-} = \int_{\boldsymbol{r}_0\cdot\boldsymbol{u}_0>0} \mathrm{d}\Gamma_0\, f_0(\Gamma_0)\eta(D - |\boldsymbol{r}_{\perp 2}(\Gamma_0, t_2)|). \tag{2.22}$$

The following notation is used in equations (2.19)–(2.22): $\boldsymbol{u}_1$, $\boldsymbol{r}_{\perp 1}$, $t_1$ and $\boldsymbol{u}_2$, $\boldsymbol{r}_{\perp 2}$, $t_2$ are the velocities, transverse coordinates and crossing positions of a plane $z = L$ by accelerated and retarded particles, respectively; the vector $\boldsymbol{r}_0$ is directed from the retarded to the accelerated particle.

Note that, for the spherically symmetric interaction potential (as for example in the case of 'Coulomb explosion' in vacuum), the numbers of accelerated and retarded particles entering the detector are identical, as follows from (2.21) and (2.22). Deflection from the spherically symmetric potential for the ion interaction in a foil results in the difference between the numbers $N^{+}$ and $N^{-}$. Therefore by measuring the difference $\Delta N = N^{+} - N^{-}$ one may judge the character of particle interaction in thin foils. Kumbartzki *et al* (1982) were the first to carry out this experiment.

Note also that equations (2.13)–(2.16) and (2.19)–(2.22) give a key to a more general problem, since they are independent of the method by which the molecular ion beam is being stripped of its electrons. These relations determine the distribution function and number of particles being detected, taking into account pair correlations, provided that the beam particle trajectories are known.

One can ignore collisions and interaction between the ions of different diclusters in equation (2.1), if $n_0^{-1/3} \gg u_0/\omega_p$, where $n_0$ is the ion beam concentration, $\omega_p$ is the plasma frequency of the foil electrons and $u_0$ is the beam velocity. Also the distance $l_0$ at which ionization of the molecular ion takes place should be less than the foil thickness $a_0$. Let us estimate the validity of these conditions according to the parameters of the ionic thermonuclear fusion. Indeed, the ion current is usually of order $I_0 \simeq 1$ MA (Zinamon 1993) and has a cross sectional area of order 1 cm$^2$. For the energy value $E_0 \simeq 20$ MeV, the molecular hydrogen beam will have a concentration equal to $n_0 \simeq 10^{15}$ cm$^{-3}$. Thus, the average distance between the ions is of the order $n_0^{-1/3} \simeq 10^3$ Å, which is much longer than the wake wavelength in metallic foils ($2\pi u_0/\omega_p \simeq 100$ Å) (Kumbartzki *et al* 1982).

The ionization distance $l_0$ may be evaluated from the known expression for ionization losses (Landau and Lifshitz 1982). Taking the ionization energy for a molecular ion equal to 10 eV, we obtain $l_0 \simeq 7$ Å.

## 3. Distribution of ions flying out from a foil in the beam direction

In this section we shall consider the propagation of a molecular ion beam through a system of two foils (as in the experiment by Kumbartzki *et al* (1982)). The second foil will be considered as a layer of plasma-like medium.

We shall consider the ions formed after the 'Coulomb explosion' to be identical. In real experiments this is true for $H_2^+$, $D_2^+$ and $He_2^+$ molecular ions (Gemmell 1980, Remillieux 1980).

Let two particles having identical masses $m_1 = m_2 = m$ and charges $Q_1 = Q_2 = Q > 0$ be interacting by a Coulomb law. Initially, at a time $t = 0$ (the instant of 'Coulomb explosion'), the relative velocity $\dot{r}$ of particles is equal to zero, while their initial distance $r$ is $r_0$. When $r \gg r_0$, the distance and relative velocity of particles at an instant $t$ are given by the following expressions: $r \simeq v_i t$ and $\dot{r} \simeq v_i = (4Q^2/mr_0)^{1/2}$; when $t \ll t_0 = r_0/v_i$, $r/r_0 \simeq 1 + (t/2t_0)^2$ (Landau and Lifshitz 1973).

For a sufficiently thin first foil, the ions passing through the foil may be considered as interacting by the Coulomb law. Then the upper limit for the foil thickness may be found to be $a_0 < 2(u_0/v_i)r_0$, where $u_0$ is the molecular beam velocity ($v_i \ll u_0$). On the other hand, the thickness $a_0$ should exceed the ionization length $l_0$ for a molecule: $a_0 > l_0$.

We shall assume that the gap $d$ between the foils is broad enough for the ions to be separated by a distance larger than $r_0$ while passing the gap. Then the following expression approximate the values of $r$ and $\dot{r}$ at the instant when ions enter the second foil: $r_c \simeq (v_i/u_0)d$ and $\dot{r}_c \simeq v_i$.

Consider now the distribution function for ions flying out from the second foil in the direction of $u_0$. For this, we shall make use of equations (2.19) and (2.20). Then, upon integration over $\boldsymbol{R}_0$ and the azimuth angle of $\boldsymbol{r}_0$, we have

$$f(u_z) = N \int_0^\infty \mathrm{d}r_0\, f_0(r_0) \int_0^{\pi/2} \mathrm{d}\theta\, \sin\theta\, \delta(u_z - u_z(\Gamma_0))\delta(\boldsymbol{u}_\rho(\Gamma_0)) \qquad (3.1)$$

where $\Gamma_0$ denotes the set of variables $r_0$ and $\theta$ (an angle between $\boldsymbol{r}_0$ and $\boldsymbol{u}_0$), and $u_z(\Gamma_0)$ and $\boldsymbol{u}_\rho(\Gamma_0)$ are the longitudinal and transverse components, respectively, of the ion velocity with respect to the $z$ axis. The following expression for the initial distribution function was used to obtain (3.1):

$$\begin{aligned} &f_0(\Gamma_0) = \frac{1}{2\pi r_0^2} f_0(r_0)\Psi(\boldsymbol{R}_0) \\ &\int \mathrm{d}\boldsymbol{R}_0\, \Psi(\boldsymbol{R}_0) = N \qquad \int_0^\infty \mathrm{d}r_0\, f_0(r_0) = 1 \end{aligned} \qquad (3.2)$$

in which $\Psi(\boldsymbol{R}_0)$ is the initial distribution function of the centre $\boldsymbol{R}_0$ of masses of diclusters, while $f_0(r_0)$ is the probability density for the cluster to have a size $r_0$.

From the axial symmetry of the problem it follows $\boldsymbol{u}_\rho(\Gamma_0) = 0$ when $\theta = 0$. Taking into account the relation $|\boldsymbol{u}_\rho(\Gamma_0)| = U(r_0)\sin\theta$ (Gorbunov and Nersissian 1993) we may evaluate the inner integral in (3.1):

$$f(u_z) = \frac{N}{2\pi} \int_0^\infty \frac{\mathrm{d}r_0}{U^2(r_0)} f_0(r_0)\delta(u_z - u_z(r_0)) \qquad (3.3)$$

where $u_z(r_0)$ is the value of $u_z(\Gamma_0)$ at $\theta = 0$. When $u_z = u_{1z}$, equation (3.3) describes the distribution function of accelerated particles and, at $u_z = u_{2z}$, the distribution function of retarded particles.

The velocity of ions flying out from the second foil in the direction of $\boldsymbol{u}_0$ is described by the expressions (see appendix) (Gorbunov and Nersissian 1993)

$$\begin{aligned} &u_{1z}(r_0) = u_0 + \frac{v_i}{2} \\ &u_{2z}(r_0) = u_0 - \frac{v_i}{2} - \frac{2(Qk_p)^2 a}{mu_0} \ln(2\mu)\exp(-\gamma r_c)\cos(k_p r_c) \end{aligned} \qquad (3.4)$$

and

$$U_1(r_0) = \frac{v_i}{2}$$
$$U_2(r_0) = \left| \frac{v_i}{2} - \frac{(\mu Q k_p)^2 a}{m u_0} \exp(-\gamma r_c) k_p r_c \sin(k_p r_c) \right| \tag{3.5}$$

where $k_p = \omega_p / u_0$, $\gamma = \nu / 2u_0$, $\mu = u_0 / v_0$, and $\omega_p$, $\nu$ and $v_0$ are the plasma frequency, efficient frequency of collisions, and average value of velocity of electrons in the second foil, respectively. ($v_0$ is equal to the Fermi velocity in metals and to the thermal velocity in plasma.) Equations (3.4) and (3.5) were obtained on the assumption that the relative distance between the ions undergoes a little change during the time of transition through the second foil ($a/u_0$). This imposes a limit on the thickness of the second foil (Gorbunov and Nersissian 1993): $a < (2/\mu k_p)[m u_0^2 / 2Q^2 k_p]^{1/2}$.

In order to evaluate the integral over $r_0$ in (3.3), we make the following observation. In real experiments where molecular ions pass through thin films, the function $f_0(r_0)$ is non-zero only in a certain interval $r_{min} \leqslant r_0 \leqslant r_{max}$ (Kanter *et al* 1980). On the other hand, equations (3.4) and (3.5) were obtained under the assumptions that $r_c > r_0$ and $u_0 \gg v_i$. Hence, we may obtain the lower and upper limits for integration over $r_0$: $4Q^2/m u_0^2 \ll r_0 < [4Q^2 d^2 / m u_0^2]^{1/3}$. We shall assume that the following inequalities are satisfied: $r_{min} \gg 4Q^2/m u_0^2$ and $r_{max} < (4Q^2 d^2 / m u_0^2)^{1/3}$. Then equations (3.4) and (3.5) are valid in the whole interval where $f_0(r_0)$ is non-zero. Using again the well known expression for the $\delta$-function (Landau and Lifshitz 1982) and passing to a new integration variable $v_i$, we shall obtain from (3.3) the following distribution function of accelerated particles:

$$f^+(u) = (N Q^2 / \pi m u^5) f_0(Q^2 / m u^2) \tag{3.6}$$

where $u = u_{1z} - u_0 > 0$.

In a similar manner, for retarded particles we obtain from (3.3)

$$f^-(u) = \frac{N Q^2}{\pi m \bar{v}^5} f_0\left(\frac{Q^2}{m \bar{v}^2}\right) \left[1 - 2\frac{(\mu Q k_p)^2 d}{m u_0^2} k_p a \exp\left(-\frac{2\gamma \bar{v} d}{u_0}\right) \sin\left(\frac{2\bar{v} k_p d}{u_0}\right)\right]^{-2} \tag{3.7}$$

where $u = u_{2z} - u_0 < 0$, while the value of $\bar{v}$ is determined from the transcendental equation

$$\bar{v} + \frac{2(Q k_p)^2 a}{m u_0} \ln(2\mu) \exp\left(-\frac{2\gamma \bar{v} d}{u_0}\right) \cos\left(\frac{2\bar{v} k_p d}{u_0}\right) = -u > 0. \tag{3.8}$$

The solution to (3.8) is unique when $B = 4Q^2 (k_p^3 a d) \ln(2\mu) / m u_0^2 \leqslant 1$, and multiple when $B > 1$. From a restriction on $a$, the following restriction on $B$ is obtained:

$$B < k_p d \frac{4 \ln(2\mu)}{\mu} \left(\frac{2Q^2 k_p}{m u_0^2}\right)^{1/2}. \tag{3.9}$$

The distribution function of retarded ions is non-zero when $Q^2 / m r_{max} \leqslant \bar{v} \leqslant Q^2 / m r_{min}$. Let $r_{max}$ satisfy another condition, namely that

$$r_{max} \ll \frac{m u_0^2}{(Q k_p)^2 [2(k_p a) \ln(2\mu)]^2}. \tag{3.10}$$

In this case, the first term in (3.8) is significantly larger than the second, so that in the square brackets in (3.7) we may use $\bar{v} = -u$. Then equation (3.8) will have the following approximate solution:

$$\bar{v} = -u - \frac{2(Q k_p)^2 a}{m u_0} \ln(2\mu) \exp\left(\frac{2\gamma u d}{u_0}\right) \cos\left(\frac{2u k_p d}{u_0}\right) \equiv h(u). \tag{3.11}$$

Substitution of this relation into (3.7) gives the final form of distribution function for the regarded particles:

$$f^-(u) = \frac{NQ^2}{\pi m}\frac{h_0(u)}{h^5(u)} f_0\left(\frac{Q^2}{mh^2(u)}\right) \tag{3.12}$$

where

$$h_0(u) = \left[1 + \frac{2(\mu Q k_p)^2 d}{m u_0^2} k_p a \exp\left(\frac{2\gamma u d}{u_0}\right)\sin\left(\frac{2uk_p d}{u_0}\right)\right]^{-2}. \tag{3.13}$$

Consider now a particular example when $f_0(r_0)$ is a Gaussian distribution. It is known from experiments (Kanter *et al* 1980) that $\bar{r}_0 > \lambda$, where $\lambda$ is the width of the Gaussian distribution and $\bar{r}_0$ is the most probable value of $r_0$. Therefore with a high degree of accuracy we may take $\langle r_0\rangle \simeq \bar{r}_0$, $r_{min} \simeq \bar{r}_0 - \lambda$ and $r_{max} \simeq \bar{r}_0 + \lambda$, where $\langle r_0\rangle$ is the average value of $r_0$.

Turning to distribution functions in which $f_0(r_0)$ is Gaussian we may observe, on the basis of equations (3.6) and (3.12), the following. First, there is a peak at the velocity value $u = (Q^2/m\bar{r}_0)^{1/2} \equiv \bar{v}_i$ of accelerated particles, and at $h(u) = \bar{v}_i$ for retarded particles. Secondly, these peaks are not symmetric with respect to $u = 0$. While for the accelerated particles the peak position is determined by the $\bar{r}_0$-value, for retarded particles the peak position oscillates as a function of the vacuum gap $d$ between the foils. The latter peak is shifted from $u = 0$ for a distance larger than $\bar{v}_i$ when the major part of the retarded particles is in a braking phase of the wake wave (i.e. when $\cos[(2u/u_0)k_p d] > 0$ in (3.11)). If the retarded particles are predominantly in the accelerating phase of the wake wave (with $\cos[(2u/u_0)k_p d] < 0$), then the peak is shifted for values less than $\bar{v}_i$. Thirdly, the peak value itself oscillates as a function of $d$ for the retarded particles. The quantity $h_0 < 1$ when the majority of retarded particles are in a defocusing phase of the wake wave (i.e. when $\sin[(2u/u_0)k_p d] > 0$ in (3.13)). When the opposite inequality holds, most of the retarded particles are in the focusing phase of the wake wave ($\sin[(2u/u_0)k_p d] < 0$). Note especially that, when the quantity $h_0 \gg 1$, the peak height of retarded particles may significantly exceed that of the accelerated particles.

In experiments with small clusters (Gemmell 1980, Remillieux 1980), the energy spectrum of particles flying out from the foil at a zero angle with respect to the initial beam direction had the following characteristics. The peak for slower particles was always a little higher than for faster particles and was shifted from $u = 0$ a little farther. The same conclusion may be made on the basis of equations (3.11)–(3.13), if we formally assume that $(2u/u_0)k_p d \ll 1$.

Finally, we shall give some numerical values. As follows from the paper of Kanter *et al* (1980), for molecular hydrogen, $\bar{r}_0 \simeq 1.2$ Å and $\lambda \simeq 0.6$ Å. Therefore $r_{min}$- and $r_{max}$-values should be $r_{min} \simeq 0.6$ Å and $r_{max} \simeq 1.8$ Å. Meanwhile with $u_0 \simeq 4.4 \times 10^9$ cm s$^{-1}$, $k_p^{-1} \simeq 14.6$ Å, $\nu/\omega_p = 0.1$, $\mu = 10$, $a = 400$ Å, $d \simeq 10$ $\mu$m we have $r_{min} \gg 2.5 \times 10^{-6}$ Å, $r_{max} \ll 29$ Å and $B < 2$.

One can conclude from these estimations that equations (3.4) and (3.5) can be used to obtain the distribution functions of the ions emitted from the foil in the direction of the vector $\boldsymbol{u}_0$. The results of such calculations are presented in figures 1 and 2.

## 4. Number of accelerated and retarded ions registered by the detector

In section 3, we have considered the effect of a wake field on the distribution function of ions flying out from the foil in the initial beam direction. It would also be interesting to

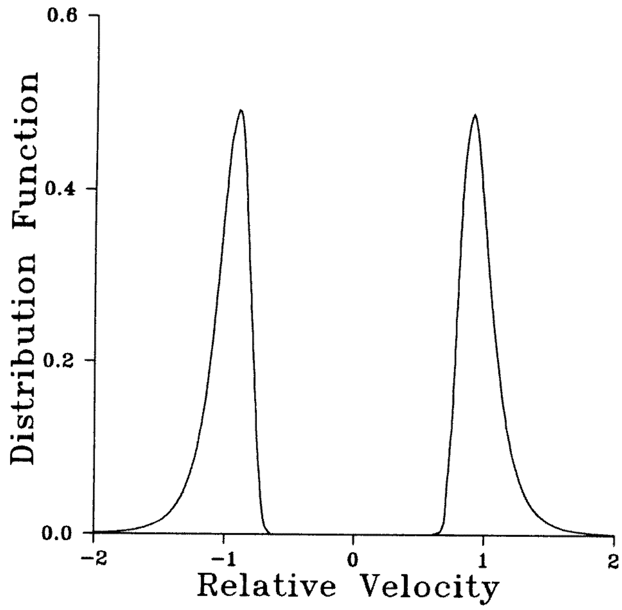


**Figure 1.** Distribution function $\bar{v}_i^3 N^{-1} f(u)$ of accelerated and retarded protons versus their relative velocity $u$ normalized to $\bar{v}_i = (Q^2/m\bar{r}_0)^{1/2}$, when the cluster size $r_c$ is less than wake wavelength ($d = 0.15$ $\mu$m). The drawings are based on equations (3.6) and (3.11) through (3.13), and on the aforementioned values of parameters as well. Peak resolution does not exceed 1% (Gemmell 1980, Remillieux 1980).

obtain the number of accelerated and retarded ions counted by the detector (Kumbartzki *et al* 1982).

Let the molecular ions be distributed homogeneously in the initial beam, the beam itself having a cylindrical form of radius $b$ and length $l$. Let also the cluster axes be homogeneously distributed within the solid angle $\mathrm{d}\Omega = \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi$ $(0 \leqslant \theta \leqslant \theta_m \ll \pi/2)$, and the relative velocity of ions be zero at the instant of 'Coulomb explosion'. In real experiments, the narrow directed beams ($\theta_m \ll \pi/2$) are obtained by the passage of diclusters through a quadrupole lens system (Kumbartzki *et al* 1982).

Let two identical ions be formed in the process of molecular ionization. Making use of the relation $\boldsymbol{r}_{\perp j}(\Gamma_0, t_j) = \boldsymbol{R}_{0\perp} + (\boldsymbol{r}_{\perp 0}/r_{\perp 0})s_j$, $j = 1, 2$ (see appendix) (Gorbunov and Nersissian 1993), where $\boldsymbol{r}_{\perp 0}$, $\boldsymbol{R}_{0\perp}$ are the transverse components of the vectors $\boldsymbol{r}_0$ and $\boldsymbol{R}_0$, $r_{\perp 0} = |r_{\perp 0}|$,

$$s_1 = (v_i t_1/2)\sin\theta \tag{4.1}$$

$$s_2 = -t_2[(v_i/2)\sin\theta - [2(Qk_p)^2 a/mu_0]\exp(-\gamma r_c\cos\theta)K(k_p r_c\sin\theta)\sin(k_p r_c\cos\theta)] \tag{4.2}$$

and integrating in (2.21) and (2.22) over the azimuthal angles of $\boldsymbol{r}_0$ and $\boldsymbol{R}_0$ (Gradshteyn and Ryzhik 1965), we may obtain the following expressions:

$$N^{\pm} = \frac{ND}{b\sin^2(\theta_m/2)} \int_0^{\infty} \mathrm{d}r_0\ f_0(r_0) \int_0^{\theta_m} \mathrm{d}\theta\ \sin\theta F(|s_j|, D, b) \tag{4.3}$$

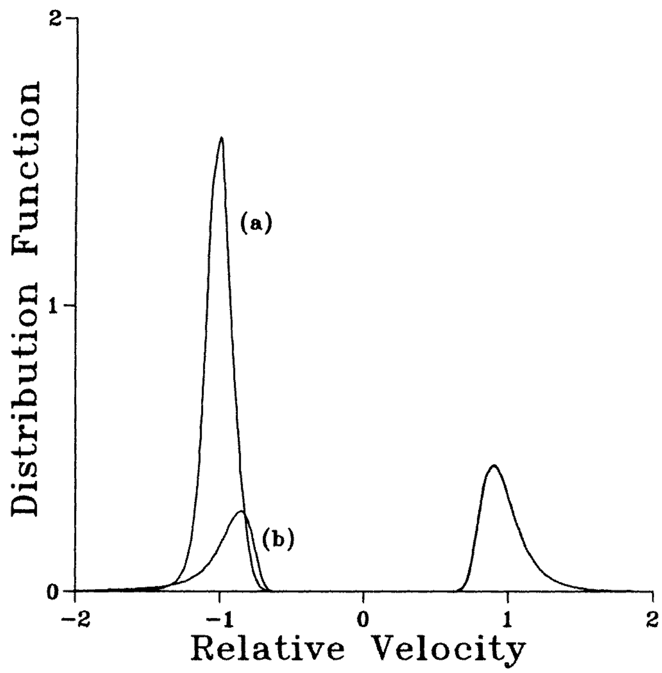


**Figure 2.** Distribution function $\bar{v}_i^3 N^{-1} f(u)$ of accelerated and retarded protons versus their relative velocity $u$ normalized to $\bar{v}_i = (Q^2/m\bar{r}_0)^{1/2}$, when the cluster size $r_c$ is greater than the wake wavelength. Curves (a) and (b) correspond to $d = 4\ \mu$m and $d = 6\ \mu$m, respectively.

where

$$F(a,\beta,\gamma) = \begin{cases} \dfrac{\min[\gamma,\beta]}{2\max[\gamma,\beta]} & \text{if } 0 < a < |\beta-\gamma| \\ \dfrac{\gamma}{2\pi\beta}\cos^{-1}\left(\dfrac{a^2+\gamma^2-\beta^2}{2a\gamma}\right) & \\ \quad +\dfrac{\beta}{2\pi\gamma}\cos^{-1}\left(\dfrac{\beta^2+a^2-\gamma^2}{2a\beta}\right) & \\ \quad -\dfrac{1}{4\pi\beta\gamma}([a^2-(\beta-\gamma)^2] & \\ \quad \times[(\beta+\gamma)^2-a^2])^{1/2} & \text{if } |\beta-\gamma| < a < \beta+\gamma \\ 0 & \text{if } a > \beta+\gamma \end{cases} \tag{4.4}$$

$$\min[a;b] = \begin{cases} a & \text{if } a \leqslant b \\ b & \text{if } a > b \end{cases} \qquad \max[a;b] = a + b - \min[a,b]. \tag{4.5}$$

In equation (4.2), $K(x) \simeq \mu^2 x/2$ at $x < 1/\mu$ and $K(x) = K_1(x)$ at $x > 1/\mu$ (see appendix) (Akopian and Matevossian 1987, Gorbunov *et al* 1992, Gorbunov and Nersissian 1993), where $K_1(x)$ is the McDonalds function.

Thus equations (4.1)–(4.5) completely determine the number of accelerated and retarded particles counted by the detector.

Equation (4.3) is significantly simplified when the transverse size of the beam is much less than the diaphragm, i.e. $b \ll D$, and when all molecules have identical size $\bar{r}_0$ (i.e. $f_0(r_0) = \delta(r_0 - \bar{r}_0)$):

$$N^{\pm} = \frac{N}{2\sin^2(\theta_m/2)} \int_0^{\theta_m} \mathrm{d}\theta \ \sin\theta \ \eta(D - |s_j|). \tag{4.6}$$

We shall now find the number of accelerated and retarded particles subject to the condition that $r_c \sin\theta_m \leqslant r_D = v_0/\omega_p$. This condition restricts the maximum size and orientation angle of the cluster: $1 < k_p r_c \leqslant 1/\mu\theta_m$, $\theta_m < 1/\mu \ll 1$.

For accelerated particles, from (4.1) and (4.6) we have $(t_1 \simeq t_2 \simeq L/u_0)$

$$\frac{N^+}{N} = \min\left\{1; \left(\frac{2u_0 D}{v_i \theta_m L}\right)^2\right\} \tag{4.7}$$

and, for retarded particles,

$$\frac{N^-}{N} = \min\left\{1; \left(\frac{2u_0 D}{v_i \theta_m L}\right)^2 \left[1 - \frac{v_i}{2u_0}\mu^2(k_p a)k_p^2 r_0 r_c \exp(-\gamma r_c)\sin(k_p r_c)\right]^{-2}\right\}. \tag{4.8}$$

In order to interpret (4.7) and (4.8) we observe that $(v_i/2u_0)\theta_m L$ is the maximum deflection of the accelerated particles from the $z$ axis at the instant of entering the detector $(\sin\theta_m \simeq \theta_m \ll 1)$. Since (4.7) includes the ratio of diaphragm diameter to maximum deflection of particles, the beam of accelerated particles will completely pass through the diaphragm when $D > \rho_m = (v_i/2u_0)\theta_m L$. If the opposite inequality holds $(D < \rho_m)$, then $N^+$ is determined by the ratio of cross section of the diaphragm to that of the beam of accelerated particles entering the detector.

The number of retarded particles is determined by the effect of the wake field. In particular, this may mean that the numbers of accelerated and retarded particles are essentially different. Consider a numerical example. For the values of parameters $v_i$, $u_0$, $\mu$, $k_p$ and $r_0$ considered in section 3 and $D = 0.1$ mm, $L = 5.13$ m, $\theta_m = 1.43^\circ$, $r_c = 54.17$ Å, $a = 1366$ Å, $\nu/\omega_p = 0.1$ (at these values, $r_c$ and $\theta_m$ have the bounds 14.6 Å $< r_c <$ 58.4 Å, $\theta_m < 5.7^\circ$), the beam of accelerated protons completely passes through the diaphragm $N^+ \simeq N$, while only 25% of the retarded protons enter the detector $(N^-/N^+ = 0.25)$.

With increases in the cluster size $r_c(r_c \sin\theta_m > r_D)$ at a fixed maximum angle $\theta_m$, the number of retarded particles entering the second foil with values of $\rho_c = r_c \sin\theta \leqslant r_D$ is decreased. Meanwhile, the number of retarded particles entering the foil with large values of $\rho_c = r_c \sin\theta \geqslant r_D$ is increased. In the first case, the retarded particle may significantly alter the direction of its motion after the foil. In the second case, the increment in the emergence angle for the retarded particle is less than the emergence angle resulting from 'Coulomb explosion'.

Let $r_D/\sin\theta_m \leqslant r_c \leqslant 1/(k_p \sin\theta_m)$. In the interval $r_D/r_c \leqslant \theta \leqslant \theta_m$ we may use the approximation for $K_1(x) \simeq 1/x$, when $1/\mu \leqslant x \leqslant 1$, to obtain from (4.2) the following expression $(\theta \ll 1)$:

$$|s_2(\theta)| = \frac{v_i}{2u_0} L \left|\theta - \frac{v_i}{u_0} k_p^2 a r_0 \exp(-\gamma r_c) \frac{\sin(k_p r_c)}{k_p r_c} \frac{1}{\theta}\right|. \tag{4.9}$$

The number of accelerated particles is almost independent of $r_c$ $(L \gg d)$ and is determined by equation (4.7). Meanwhile, the number of retarded particles that have originated from clusters with initial orientation $0 \leqslant \theta \leqslant r_D/r_c$ and $r_D/r_c \leqslant \theta \leqslant \theta_m$ is determined by the following expressions:

$$N_1^- \simeq (2N/\theta_m^2) \int_0^{r_D/r_c} \mathrm{d}\theta\, \theta \eta(D - |s_2(\theta)|) \tag{4.10}$$

$$N_2^- \simeq (2N/\theta_m^2) \int_{r_D/r_c}^{\theta_m} \mathrm{d}\theta\, \theta \eta(D - |s_2(\theta)|). \tag{4.11}$$

In equation (4.10), $K(x) \simeq \mu^2 x/2$ and, in (4.11), $s_2(\theta)$ is determined by equation (4.9).

Evaluation of the integrals (4.10) and (4.11) gives

$$\frac{N_1^-}{N} = \min\left\{(\mu\theta_m k_p r_c)^{-2}; \left(\frac{2u_0 D}{v_i \theta_m L}\right)^2 \left[1 - \frac{v_i}{2u_0}\mu^2(k_p a)(k_p^2 r_0 r_c \exp(-\gamma r_c)\sin(k_p r_c)\right]^{-2}\right\} \tag{4.12}$$

$$\frac{N_2^-}{N} = \begin{cases} 0 & \text{if } \sin(k_p r_c) < 0 \text{ and } \dfrac{u_0 D}{v_i L} < \left(\dfrac{v_i}{u_0} k_p^2 a r_0 \exp(-\gamma r_c) \dfrac{|\sin(k_p r_c)|}{k_p r_c}\right)^{1/2} \\ \tilde{\theta}_2^2 - \tilde{\theta}_1^2 & \text{if } \sin(k_p r_c) > 0 \\ \tilde{\theta}_2^2 - \tilde{\theta}_1^2 & \text{if } \sin(k_p r_c) < 0 \text{ and } \dfrac{u_0 D}{v_i L} > \left(\dfrac{v_i}{u_0} k_p^2 a r_0 \exp(-\gamma r_c) \dfrac{|\sin(k_p r_c)|}{k_p r_c}\right)^{1/2} \end{cases} \tag{4.13}$$

where the following notation was used:

$$\tilde{\theta}_2 = \min(1; \theta_2/\theta_m) \qquad \tilde{\theta}_1 = \max[(\mu\theta_m k_p r_c)^{-1}; \theta_1/\theta_m] \tag{4.14}$$

$$\tilde{\theta}_2 = \tilde{\theta}_1 \qquad \text{if } \theta_2 < (\mu k_p r_c)^{-1} \text{ or } \theta_1 > \theta_m \tag{4.15}$$

$$\theta_1 = S\left\{\left\{\left(\frac{u_0 D}{v_i L}\right)^2 + \frac{v_i}{u_0} k_p^2 a r_0 \exp(-\gamma r_c) \frac{\sin(k_p r_c)}{k_p r_c}\right\}^{1/2} - \frac{u_0 D}{v_i L}\right\} \tag{4.16}$$

$$\theta_2 = \frac{u_0 D}{v_i L} + \left\{\left(\frac{u_0 D}{v_i L}\right)^2 + \frac{v_i}{u_0} k_p^2 a r_0 \exp(-\gamma r_c) \frac{\sin(k_p r_c)}{k_p r_c}\right\}^{1/2}. \tag{4.17}$$

Here $S = \mathrm{sgn}[\sin(k_p r_c)]$, $\theta_1$ and $\theta_2$ are the values of $\theta$ at which the retarded particle at the instance of entering the diaphragm is at a distance $D$ from the $z$ axis.

As follows from equation (4.12), the number of retarded particles entering the second foil with small values of $\rho_c (< r_D)$ becomes smaller when the cluster size increases (or when the vacuum gap $d$ becomes larger). For large values of $r_c$ satisfying the conditions

$$\left(\frac{u_0 D}{v_i L}\right)^2 \gg \frac{v_i}{u_0} k_p^2 a r_0 \frac{\exp(-\gamma r_c)}{k_p r_c} \qquad r_c < (k_p \theta_m)^{-1} \tag{4.18}$$

equations (4.16) and (4.17) are significantly simpler. In that limiting case we have

$$\theta_1 = \frac{v_i^2}{2u_0^2} \frac{L}{D} k_p^2 a r_0 \exp(-\gamma r_c) \frac{|\sin(k_p r_c)|}{k_p r_c} \tag{4.19}$$

$$\theta_2 = \frac{2u_0 D}{v_i L} + \frac{v_i^2}{2u_0^2} \frac{L}{D} k_p^2 a r_0 \exp(-\gamma r_c) \frac{\sin(k_p r_c)}{k_p r_c}. \tag{4.20}$$

Note from (4.19) and (4.20) that $\theta_1 \ll \theta_2$. The number of retarded particles having large values of $\rho_c (> r_D)$ may be obtained in the practically important case when

$\rho_m(v_i/2u_0)\theta_m L \simeq D$ (Kumbartzki *et al* 1982). From (4.14) it follows that $\tilde{\theta}_1 \ll \tilde{\theta}_2 \simeq 1$, so that the number of retarded particles is determined by a term proportional to $\tilde{\theta}_2^2$:

$$\frac{N^-}{N} \approx \tilde{\theta}_2^2 \approx \min\left\{1; \left(\frac{2u_0 D}{v_i\theta_m L}\right)^2 + \frac{2v_i}{u_0\theta_m^2} k_p^2 a r_0 \exp(-\gamma r_c) \frac{\sin(k_p r_c)}{k_p r_c}\right\}. \tag{4.21}$$

Kumbartzki *et al* (1982) consider also the relative variation in the number of retarded particles:

$$P = \frac{N^- - N^+}{N^+}. \tag{4.22}$$

For two values of maximum deflection $\rho_m$ of the accelerated particles, we obtain from (4.7), (4.21) and (4.22) the following relations.

(i) For $\rho_m > D$ $(\theta_m > 2u_0 D/v_i L)$,

$$P = \min\left\{\left(\frac{\rho_m}{D}\right)^2 - 1; \frac{1}{2}\left(\frac{v_i}{u_0}\right)^3 \left(\frac{L}{D}\right)^2 k_p^2 a r_0 \exp(-\gamma r_c) \frac{\sin(k_p r_c)}{k_p r_c}\right\}. \tag{4.23}$$

(ii) For $\rho_m < D$ $(\theta_m < 2u_0 D/v_i L)$,

$$P = \min\left\{0; \left(\frac{D}{\rho_m}\right)^2 - 1 + \frac{2v_i}{u_0\theta_m^2} k_p^2 a r_0 \exp(-\gamma r_c) \frac{\sin(k_p r_c)}{k_p r_c}\right\}. \tag{4.24}$$

In particular, when $\rho_m = D$ from (4.23) and (4.24) we obtain

$$P = \min\left\{0; \frac{2v_i}{u_0\theta_m^2} k_p^2 a r_0 \exp(-\gamma r_c) \frac{\sin(k_p r_c)}{k_p r_c}\right\}. \tag{4.25}$$

One may see from equations (4.23)–(4.25) and condition (4.18) that the relative variation in retarded particles oscillates as a function of vacuum gap ($r_c = (v_i/u_0)d$), with amplitude much smaller than unity. By a corresponding choice of parameters (see (4.24)), the number of retarded particles may be decreased. This takes place because the beam of accelerated particles completely passes through the diaphragm, and their number is not less than the number of retarded particles. This was, in particular, the pattern observed in the experiment by Kumbartzki *et al* (1982).

Numerically, for the parameters given above and cluster size $r_c \simeq 580$ Å ($d \simeq 38.7$ $\mu$m), the relative variance of retarded particles is 10%.

We observe in conclusion that a proper choice of the vacuum gap $d$ between two thin foils may increase the effect of the wake field on retarded particles. Thus, for example, when $r_c \simeq 54.2$ Å ($d \simeq 3.6$ $\mu$m), the number of retarded particles is a quarter of the number of accelerated particles. Meanwhile, when $r_c \simeq 580$ Å, the number of retarded particles is only 10% less than that of accelerated particles.

## Appendix

A brief discussion is given of a method by which equations (3.4), (3.5), (4.1) and (4.2) of this paper were obtained as well as the limitation imposed on the second foil thickness.

To describe the dynamics of a dicluster of charged particles passing through a thin foil, one has to determine the electric potential created by particles in the medium. For a charge $Q$ moving with velocity $\boldsymbol{u}_0$ in a homogeneous medium characterized by dielectric permeability $\varepsilon(\boldsymbol{k}, \omega)$ the potential is given by the following relation (Landau and Lifshitz 1982):

$$\varphi(\boldsymbol{r}, t) = \frac{4\pi Q}{(2\pi)^3} \int \mathrm{d}\boldsymbol{k} \frac{\exp[\mathrm{i}\boldsymbol{k} \cdot (\boldsymbol{r} - \boldsymbol{u}_0 t)]}{k^2 \varepsilon(\boldsymbol{k}, \boldsymbol{k}\boldsymbol{u}_0)}. \tag{A.1}$$

Usually, the dielectric permeability of a cold plasma is used to evaluate $\varphi(\boldsymbol{r}, t)$ (Gorbunov *et al* 1992). However, the expression thus obtained has a singularity on the particle trajectory.

In order to obtain the correct expression, taking into account the thermal motion of electrons in the medium, Akopian and Matevossian suggested a model of dielectric permeability which is identical with the static expression when $\boldsymbol{k} \cdot \boldsymbol{u}_0 < k v_0$ and with the dynamic expression when $\boldsymbol{k} \cdot \boldsymbol{u}_0 > k v_0$ ($v_0$ is the same as in section 3). Their $\varphi(\boldsymbol{r}, t)$ expression is very close to the accurate values of the potential (Wang *et al* 1981). It is also identical with the expression of Gorbunov *et al* (1992) at a distance from the particle trajectory exceeding $v_0/\omega_p$ (equal to the Debye radius for plasma, or to the Thomas–Fermi distance for a metal).

Consider now the dynamics of the dicluster formed after the Coulomb explosion of the molecular ion and entering the second foil (it is assumed, as already mentioned above, that the inter-particle distance, at the instant when they enter the second foil, exceeds the wave wavelength). Relative motion ($\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2$) of the dicluster in the foil as well as the motion of its centre $\boldsymbol{R}$ of mass are described by the following equations (Kagan *et al* 1978):

$$\ddot{\boldsymbol{r}} = \frac{q_1 q_2}{\mu_0 r^2} \frac{\boldsymbol{r}}{r} + \frac{\boldsymbol{f}_2^s}{m_1} - \frac{\boldsymbol{f}_2^s}{m_2} + \frac{1}{m_1} \boldsymbol{f}_{21}(\boldsymbol{r}) - \frac{1}{m_2} \boldsymbol{f}_{12}(\boldsymbol{r}) \tag{A.2}$$

$$(m_1 + m_2)\ddot{\boldsymbol{R}} = \boldsymbol{f}_1^s + \boldsymbol{f}_2^s + \boldsymbol{f}_{21}(\boldsymbol{r}) + \boldsymbol{f}_{12}(\boldsymbol{r}) \tag{A.3}$$

where $\mu_0$ is the dicluster reduced mass, $\boldsymbol{f}_1^s$ and $\boldsymbol{f}_2^s$ are forces causing retardation of the first and second particle (which determine the usual polarization losses), and $\boldsymbol{f}_{21}(\boldsymbol{r})$ and $\boldsymbol{f}_{12}(\boldsymbol{r})$ are the wake forces acting on the first and second particles, respectively. Note that the general expressions for these forces are obtained from the electric potential (A.1). Analysis of each term in the right-hand side of equations (A.2) and (A.3) has shown, however, that the last terms are most significant (Akopian and Matevossian 1987, Gorbunov and Nersissian 1993), so that we have

$$\ddot{\boldsymbol{r}} = -\frac{1}{m_2} \boldsymbol{f}_{12}(\boldsymbol{r}) \qquad (m_1 + m_2)\ddot{\boldsymbol{R}} = \boldsymbol{f}_{12}(\boldsymbol{r}) \tag{A.4}$$

under the assumption that $\boldsymbol{r}_2$ is the retarded particle position vector. Equations (A.4) are easily solved under the assumption that the second foil weakly affects the relative vector $\boldsymbol{r}$. If we assume that at the instant of entering the second foil $\boldsymbol{r} = \boldsymbol{r}_c$ and $\dot{\boldsymbol{r}} = \dot{\boldsymbol{r}}_c$, then $\boldsymbol{f}_{12}(\boldsymbol{r})$ may be expanded in series of the difference $\boldsymbol{r} - \boldsymbol{r}_c$. Preserving only the linear term, one obtains the equations for coupled oscillators whose solutions have the form

$$\boldsymbol{r}(t) = \boldsymbol{r}_c + \frac{\dot{\boldsymbol{r}}_c}{\omega} \sin(\omega t) + \boldsymbol{g}[1 - \cos(\omega t)] \tag{A.5}$$

$$\dot{\boldsymbol{r}}(t) = \dot{\boldsymbol{r}}_c \cos(\omega t) + \omega \boldsymbol{g} \sin(\omega t). \tag{A.6}$$

Here the vector $\boldsymbol{g}$ is determined from the matrix equation $A_{ij} g_j = -f_{12i}(\boldsymbol{r}_c)$, $A_{ij} = \partial f_{12i}(\boldsymbol{r}_c)/\partial x_{cj}$, in which $g_i$, $f_{12i}(\boldsymbol{r}_c)$, $x_{ci}$ are components of $\boldsymbol{g}$, $\boldsymbol{f}_{12}(\boldsymbol{r}_c)$ and $\boldsymbol{r}_c$, while $\omega$ is the eigenvalue determined from the characteristic equation

$$\det\left[\frac{1}{m_2} A_{ij} - \omega^2 \delta_{ij}\right] = 0 \tag{A.7}$$

with $\delta_{ij}$ denoting the Kronecker delta.

As follows from the simplified equations (A.5) and (A.6), the variation in $\boldsymbol{r}$ is small when $\omega t < 1$, i.e. the time of dicluster propagation through the second foil is smaller than the interaction time between the particles. Hence one may obtain the limitation on the second foil thickness which was mentioned in the paper. Upon further expansion of harmonic functions in (A.5) and (A.6) in power series of $\omega t$ and retaining only quadratic terms in (A.5), the following expressions for the retarded particle coordinates and velocity may be obtained:

$$\boldsymbol{r}_2(t) = \boldsymbol{r}_{2c} + \dot{\boldsymbol{r}}_{2c} t + \frac{t^2}{2m_2} \boldsymbol{f}_{12}(\boldsymbol{r}_c) \tag{A.8}$$

$$\dot{\boldsymbol{r}}_2(t) = \dot{\boldsymbol{r}}_{2c} + \frac{t}{m_2} \boldsymbol{f}_{12}(\boldsymbol{r}_c) \tag{A.9}$$

where $\boldsymbol{r}_{2c}$ and $\dot{\boldsymbol{r}}_{2c}$ are initial values at the instant when the retarded particle enters the second foil.

The accelerated particle motion is almost unaffected by the second foil when $\omega t < 1$. This may be explained by the fact that only the monotonic component of the retarded particle wake potential affects the motion of the accelerated particle, while the retarded particle feels the effect of a long-range oscillating wake potential of the accelerated particle, which is significantly stronger than the monotonic field component (under the imposed limitations).

Thus, if the potential of a particular medium is known, one may obtain the force $\boldsymbol{f}_{12}(\boldsymbol{r})$. In the case when $r_c \sin\theta < v_0/\omega_p$, the expression for this force may be obtained from the potential found by Akopian and Matevossian (1987), while Gorbunov *et al* (1992) have found the potential in the opposite case $r_c \sin\theta > v_0/\omega_p$. Thus the force obtained was used to determine equations (3.4), (3.5), (4.1) and (4.2).

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